No. of printed pages:15

CLASS: XII

MID TERM EXAM (SESSION: 2023– 24)

SUBJECT: APPLIED MATHEMATICS (CODE 241)

SOLUTION SET/ MARKING SCHEME – SET B

Time Allowed: 3 Hrs. **Max. Marks: 80**

Set B	Expected Answers/ Value Points	Marks
	SECTION A (Each question carries 1-mark weightage)	
Q1.	The solution of the differential equation $\frac{dx}{x}+\frac{dy}{y}=0$ is (a) $\frac{1}{x}+\frac{1}{y}=C$ (b) $x+y=C$ (c) $logx\ logy=C$ (d) $xy=C$ Sol. (d) $xy=C$	1
Q2.	If $\int_0^{40}\frac{dx}{2x+1}=\log k$, then the value of k is (a) 4 (b) 3 (c) 9 (d) 1 Sol. (c) 9	1
Q3.	If the radius of a circle is increasing at the rate of 2 cm/sec, then the area of the circle when its radius is 20 cm is increasing at the rate of (a) $80\pi\ m^2/sec$ (b) $80\ m^2/sec$ (c) $80\pi\ cm^2/sec$ (d) $80\ cm^2/sec$ Sol. (c) $80\pi\ cm^2/sec$	1
Q4.	$\int\frac{(\log x)^5}{x}dx$ is equal to (a) $\frac{\log x^6}{6}+C$ (b) $\frac{(\log x)^6}{3x^2}+C$ (c) $\frac{\log x^6}{3x^2}+C$ (d) $\frac{(\log x)^6}{6}+C$ Sol. (d) $\frac{(\log x)^6}{6}+C$	1
Q5.	If $\frac{x+1}{x+2}\geq 1, then$ (a) $x\in[-\infty,2]$ (b) $x\in(-\infty,-2)$ (c) $x\in(-\infty,2]$ (d) $x\in(-\infty,2)$ Sol. (b) $x\in(-\infty,-2)$	1
Q6.	A vehicle costing Rs. 125000 has scrap value of Rs. 25000. If annual depreciation charge is Rs. 12500, then useful life of the vehicle is (a) 4 years (b) 6 years (c) 8 years (d) 10 years Sol. (c) 8 years	1
Q7.	$\int e^x\left(\frac{1}{x^2}-\frac{2}{x^3}\right)dx$ is equal to (a) $\frac{e^x}{x^2}+C$ (b) $\frac{e^x}{x^3}+C$ (c) $\frac{-e^x}{x^2}+C$ (d) $\frac{e^x}{2x^3}+C$	

	Sol. $\frac{e^x}{x^2} + C$	1
Q8.	If $\mathrm{C}(x)$ and $\mathrm{R}(x)$ are respectively Cost function and Revenue function, then profit function $\mathrm{P}(x)$ is given by (a) $P(x) = R(x)$ (b) $P(x) = C(x) + R(x)$ (c) $P(x) = R(x) - C(x)$ (d) $P(x) = R(x).C(x)$ Sol. $P(x) = R(x) - C(x)$	1
Q9.	The least non-negative remainder, when 3^{15} is divided by 7 is (a) 5 (b) 1 (c) 6 (d) 7 Sol. (c) 6	1
Q10.	The equation of normal at the point (1,1) to the curve $2y + x^2 = 3$ is (a) $x + y = 0$ (b) $x - y = 0$ (c) $x + y = 1$ (d) $x - y = 1$ Sol. (b) $x - y = 0$	1
Q11.	The ratio in which a grocer mixes two varieties of pulses costing Rs. 85 per kg and Rs. 100 per kg respectively so as to get a mixture worth Rs. 92 per kg is (a) $8:7$ (b) $7:8$ (c) $5:7$ (d) $7:5$ Sol. $8:7$	1
Q12.	For what value of x, is the following matrix singular? $$\begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$$ (a) $x = -1$ (b) $x = 2$ (c) $x = 1$ (d) $x = -2$ Sol. (c) $x = 1$	1
Q13.	Region represented by $x \geq 0, y \geq 0$ lies in (a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant Sol. I quadrant	1
Q14.	Let $p > 0$ and $q < 0$ and $p, q \in Z$, then choose the correct inequality from the given below options to complete the statement: $p + q$ ____ $p - q$ (a) $>$ (b) $\leq$ (c) $\geq$ (d) $<$ Sol. (d) $<$	1
Q15.	If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, then the values of x and y are: (a) $x = 7, y = -1$ (b) $x = -7, y = -1$ (c) $x = 7, y = 1$ (d) $x = -7, y = 1$ Sol. $x = 7, y = -1$	1
Q16.	If the value of the objective function Z can be increased or decreased indefinitely, such solution is called…………. (a) Bounded solution (b) Optimum solution (c) Unbounded solution (d) Feasible solution Sol. (c) Unbounded solution	1

Q17.	An investment of ₹ 5,000 becomes ₹ 25,000 in 4 years, then the CAGR (compound annual growth rate) is given by – (a) $\left[\sqrt[4]{5}-1\right]\times 100$ (b) $\left[\sqrt[4]{5}+1\right]\times 100$ (c) $\frac{\sqrt[4]{5}-1}{100}$ (d) $\frac{\sqrt[4]{5}+1}{100}$ Sol. (a) $\left[\sqrt[4]{5}-1\right]\times 100$	1
Q18.	A and B are square matrices each of order 3 such that $\lvert A\rvert=-1$ and $\lvert B\rvert=3$. What is the value of $\lvert 3AB\rvert$? (a) -27 (b) -9 (c) -18 (d) -81 Sol. (d) -81	1

For questions 19 and 20 two statements are given – one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to the question from the codes (i), (ii), (iii) and (iv) as given below:

(i) Both A and R are true and R is the correct explanation of the assertion
(ii) Both A and R are true but R is not the correct explanation of the assertion
(iii) A is true, but R is false
(iv) A is false, but R is true

Q19.	<u>Assertion</u> (A): $\int_2^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}dx=\frac{1}{2}$ <u>Reason</u> (R): $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$ Sol. (i)	1
Q20.	<u>Assertion</u> (A): The present value of a sequence of payments of Rs. 600 made at the end of each quarter and continuing forever, if money is worth 6% compounded quarterly is Rs. 10,000. <u>Reason</u> (R): The present value of a perpetuity of Rs. R payable at the end of each period, the first being due one period hence is $P=\frac{R}{i}$ where R denotes size of each payment and i denotes rate per period. (a) (i) (b) (ii) (c) (iii) (d) (iv) Sol. (d) (iv)	1

SECTION B

(Each question carries 2-mark weightage)

Q21.	At what rate of interest will the present value of a perpetuity of Rs 500 payable at the end of every 6 months be Rs. 10,000? Sol. Let the rate of interest be $r\%$ per annum, then $i=\frac{r}{200}$ R = 500, P = 10,000 P = R/i r = 10% per annum **OR** Find the effective rate of interest equivalent to a nominal rate of 6% compounded semi-annually.	1 1

	Sol. When compounded semi-annually We have r = 0.06 and m = 2 $$r_{eff} = \left(1+\frac{r}{m}\right)^m - 1 = \left(1+\frac{0.06}{2}\right)^2 - 1 = 0.0609 = 6.09\%$$	1 1
Q22.	In a 200-metre race, Anuj can beat Param by 5 metre or 3 seconds. How much time did Anuj take to complete the race? Sol. Param runs 5 m in 3 seconds ⇒ time taken to run 200 m = 3 /5 × 200 = 120 seconds Anuj ‘s time = 120 – 3 = 117 seconds **OR** Find the remainder when (226 × 369 × 122 × 461 × 1025) is divided by 8. Sol. As we know that ((A × B) mod C = (A mod C × B mod C) mod C) ….. (i) Therefore, for calculating (226 × 369 × 122 × 461 × 1025) mod 8, let us find 226 mod 8 = 2 369 mod 8 = 1 122 mod 8 = 2 461 mod 8 = 5 1025 mod 8 = 1 using equation (i) repeatedly we have: (226 × 369 × 122 × 461 × 1025) mod 8 = (2 x 1 x 2 x 5 x 1) mod 8 = 20 (mod 8) = 4 Hence remainder is 4	 1 1 1 1
Q23.	Find the least value of a so that the function $f(x) = x^2 + ax + 1$ is strictly increasing on [1,2]. Sol. $f(x) = x^2 + ax + 1$ $f'(x) = 2x + a$ Now $1 \le x \le 2 \Rightarrow 2 \le 2x \le 4 \Rightarrow 2 + a \le 2x + a \le 4 + a$ For $f(x)$ to be strictly increasing on [1,2], $2 + a \ge 0 \Rightarrow a \ge -2$ Hence the least value of a = -2.	 1+1
Q24.	If $X = \begin{bmatrix} -1 & 3 \\ 8 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} -5 & 1 \\ -1 & -2 \end{bmatrix}$ then find the matrix Z such that $2X + Y = 5Z$. Sol. Let $Z = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $$2\begin{bmatrix} -1 & 3 \\ 8 & 4 \end{bmatrix} + \begin{bmatrix} -5 & 1 \\ -1 & -2 \end{bmatrix} = 5\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$ $$Z = \begin{bmatrix} -\frac{7}{5} & \frac{7}{5} \\ 3 & \frac{6}{5} \end{bmatrix}$$	 1 1

Q25.	A company has two groups of inspectors namely, group A and B, who are assigned to do a quality inspection work. It is required that at least 1800 pieces are inspected for 8-hour day. It is known that inspectors of group A can check pieces at the rate of 25 per hour with an accuracy of 98%, while inspectors of group B can check at the rate of 15 pieces per hour with an accuracy of 95%. The inspectors of group A and B are paid Rs 40 and Rs 30 per hour respectively to do the work. Each time an error is caused by the any inspector, it costs a loss of Rs 20 to the company. The company has 8 inspectors in group A and 10 in group B. The company wants to determine the optimal assignment of Inspectors to minimize total inspection cost. Formulate an LPP. Sol. Let x *and* y denote the number of group A and group B inspectors that may be assigned the job of quality control inspection.	1 1

	Group A Inspector	Group B Inspector
Number of Inspectors	8	10
Rate of checking per hour	25 pieces	15 pieces
Inaccuracy in checking	1-0.98=0.02	1-0.95=0.05
Cost of Inaccuracy in checking	Rs. 20	Rs. 20
Wage rate per hour	Rs. 40	Rs. 30

Cost of group A inspector per hour is
$40 + 20 \times 0.02 \times 25 = Rs. 50$
Cost of group B inspector per hour is
$30 + 20 \times 0.05 \times 15 = Rs. 45$
LPP is
Minimize $Z = 8 \times 50x + 8 \times 45y$
$Z = 400x + 360y$
Subject to constraints
$x \geq 0, y \geq 0$
$x \leq 8, y \leq 10$
$8(25x) + 8(15y) \geq 1800$ i.e. $5x + 3y \geq 45$

SECTION C

(Each question carries 3-mark weightage)

Q26.	A cistern can be filled by two pipes A and B in 12 minutes and 15 minutes respectively. Another tap C can empty the full tank in 20 minutes. If the tap C is opened 5 minutes after the pipes A and B are opened, find when the cistern becomes full?	
	Sol. Cistern filled by A and B in 5 minutes = $5\left(\frac{1}{12} + \frac{1}{15}\right) = \frac{3}{4}$	1
	Unfilled part of the tank = $\frac{1}{4}$	1
	Portion of cistern filled by A, B and C in 1 minute = $\frac{1}{12} + \frac{1}{15} - \frac{1}{20} = \frac{1}{10}$	1
	So, $\frac{1}{4}$ of the tank will be filled in = $\frac{1}{4} \times 10 = 2\ mins\ 30\ secs$ Total time taken to fill the tank = 5 + 2 mins 30 secs = 7 mins 30 secs **OR**	

	A boat covers 32 km upstream and 36 km downstream in 7 hours. Also it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream. **Sol.** Let the speed of the boat in still water be $x\ km/hr$ and the speed of the stream be y km/hr. $$\frac{32}{x-y}+\frac{36}{x+y}=7$$ $$\frac{40}{x-y}+\frac{48}{x+y}=9$$ By solving the above two equations we get $$x=10, y=2$$ Speed of the boat in still water = 10 km/hr and speed of the stream = 2 km/hr.	1 1 1
Q27.	Verify that y= $\frac{1}{x}-logx$ is a solution of the differential equation, $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=\log x$ Sol. $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=\log x$ Differentiating both sides w.r.t. x, we get $$\Rightarrow \frac{dy}{dx}=-\frac{1}{x^2}-\frac{1}{x}$$ Differentiating again both sides wrt x, we get $$\frac{d^2y}{dx^2}=\frac{2}{x^3}+\frac{1}{x^2}$$ $$LHS=\ x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y$$ Substitute the above values in LHS, we get $$LHS=\log x=RHS$$ Hence verified	1 1 1
Q28.	A machine costing Rs.2,00,000 has effective life of 7 years and its scrap value is Rs.30,000. What amount should the company put into a sinking fund earning 5% per annum, so that it can replace the machine after its useful life? Assume that a new machine will cost Rs.3,00,000 after 7 years. (Given $(1.05)^7=1.407$) Sol. Amount needed after 7 years = 3,00,000 – 30,000 = 2,70,000 The payments into sinking fund consisting of 7 annual payments at the rate 5% per year is given by $$A=R\left[\frac{(1+i)^n-1}{i}\right]$$ $$270000=R\left[\frac{(1.05)^7-1}{0.05}\right]$$ $$R=\frac{270000\times 0.05}{(1.05)^7-1}=\frac{13500}{1.407-1}=\frac{13500}{0.407}=33169.53$$	1 1 1
Q29.	If $x=\frac{1}{2}(e^t+e^{-t})$ and $y=\frac{1}{2}(e^t-e^{-t})$, show that $y^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$. Sol. Given $x=\frac{1}{2}(e^t+e^{-t})$ and $y=\frac{1}{2}(e^t-e^{-t})$ Differentiating both with respect to t, we get	

	$\frac{dx}{dt} = \frac{1}{2}(e^{t} - e^{-t})$ $\Rightarrow \frac{dx}{dt} = y$ and $\frac{dy}{dt} = \frac{1}{2}(e^{t} + e^{-t})$ $\Rightarrow \frac{dy}{dt} = x$ $\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y}$. Differentiating w.r.t.x, we get $\frac{d^2y}{dx^2} = \frac{y.1 - x.\frac{dy}{dx}}{y^2}$ $\Rightarrow y^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$	1 1 1
Q30.	Express $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. Sol. Let A = $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ $A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ $P = \frac{1}{2}(A + A')$ $P = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}$ $P' = P$ $Q = \frac{1}{2}(A - A') = \begin{pmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{pmatrix}$ $Q' = -Q$ Also $P + Q = A$	1 1 1
Q31.	The demand and supply functions are $p_d = 25 - x^2$ *and* $p_s = 2x + 1$ respectively. Find the producer's surplus at equilibrium price. Sol. $p_d = 25 - x^2$ $p_s = 2x + 1$ At equilibrium, demand = supply $\Rightarrow 25 - x_0^2 = 2x_0 + 1$	

	By solving the above equation, we get $x_0 = -6 \; and \; x_0 = 4$ $\therefore \; x_0 = 4$ (reject the negative value) $p_0 = 25 - 4^2 = 9$ $PS = 4 \times 9 - \int_0^4 (2x+1)\,dx = 36 - (2\frac{x^2}{2} + x)\Big\|_0^4 = 16$ **OR** Evaluate $\int_0^2 x^2\sqrt{2-x}\,dx$ Sol. $\int_0^2 x^2\sqrt{2-x}\,dx = \int_0^2 (2-x)^2\sqrt{2-(2-x)}\,dx$ $= \int_0^2 (4-4x+x^2)\sqrt{x}\,dx = \int_0^2 \left(4x^{\frac{1}{2}} - 4x^{\frac{3}{2}} + x^{\frac{5}{2}}\right)dx$ $= \left[4.\frac{x^{3/2}}{\frac{3}{2}} - 4.\frac{x^{5/2}}{\frac{5}{2}} + \frac{x^{7/2}}{\frac{7}{2}}\right]_0^2 = \left[x^{3/2}\left(\frac{8}{3} - \frac{8}{5}x + \frac{2}{7}x^2\right)\right]_0^2$ $= 2\sqrt{2}\left(\frac{8}{3} - \frac{8}{5}.2 + \frac{2}{7}.4\right) - 0 = 2\sqrt{2}\left(\frac{8}{3} - \frac{16}{5} + \frac{8}{7}\right)$ $= 16\sqrt{2}\left(\frac{1}{3} - \frac{2}{5} + \frac{1}{7}\right) = 16\sqrt{2} \times \frac{8}{105} = \frac{128\sqrt{2}}{105}$.	1 1 1 1+1+1
SECTION D (Each question carries 5-mark weightage)		
Q32.	A metal box with a square base and vertical sides is to contain $1024\ cm^3$ of water. The material for the top and bottom costs Rs. 5 per cm^2 and the material for the sides costs Rs. 2.50 per cm^2. Find the least cost of the box. Sol. Take x, x, y as the length, breadth and height of the tank. Thus, $x^2\,y = 1024$. Let C be the total cost of box, $C(x) = 5\,x^2 + 5\,x^2 + 2.50(4\,xy) = 10x^2 + 10xy = 10x^2 + 10x\left(\frac{1024}{x^2}\right) = 10x^2 + \frac{10240}{x}$. $\frac{dC}{dx} = 20x - \frac{10240}{x^2}$ and $\frac{d^2C}{dx^2} = 20 + \frac{20480}{x^3}$. For the maximum or minimum, we should have $\frac{dC}{dx} = 0 \Rightarrow x^3 = \frac{10240}{20} = 512$ $\Rightarrow x = 8$. $\frac{d^2C}{dx^2}\Big]_{x=8} > 0 \Rightarrow C \text{ is minimum at } x = 8.\ \ y = \frac{1024}{x^2} = \frac{1024}{64} = 16$ The minimum cost is $C = 10x^2 + 10xy = 10(8)^2 + 10(8)(16) = 1920$. Thus, the minimum cost is Rs 1920 **OR**	 1 1 1 1+1

	For a monopolist's product, the demand function is $p = \frac{50}{\sqrt{x}}$ and average cost function $AC = 0.5 + \frac{2000}{x}$. Find the profit maximizing level of output. At this level, show that the marginal revenue and marginal cost are equal. Sol. $R(x) = px = 50\sqrt{x}$ $C(x) = 0.5x + 2000$ $P(x) = R(x) - C(x) = 50\sqrt{x} - 0.5x - 2000$ $\frac{d}{dx}\left(P(x)\right) = \frac{25}{\sqrt{x}} - \frac{1}{2}$ $\frac{d^2}{dx^2}\left(P(x)\right) = -\frac{25}{2x^{3/2}}$ For maximum profit, $\frac{d}{dx}\left(P(x)\right) = 0$ $\frac{25}{\sqrt{x}} - \frac{1}{2} = 0 \Rightarrow x = 2500$ $\frac{d^2}{dx^2}\left(P(x)\right)\ at\ x = 2500 < 0$ P(x) is maximum when x=2500 $MR = \frac{d}{dx}\left(R(x)\right) = \frac{25}{\sqrt{x}}$ $MC = \frac{d}{dx}\left(C(x)\right) = 0.5$ $At\ x = 2500, MR = MC = \frac{1}{2}$	1 1 1 1 1
Q33.	The amount of radiocarbon present after t years is given by $A = A_0 e^{-(\log 2)\left(\frac{1}{5700}\right)t}, where\ A_0$ is the amount present in the living plants and animals. (i) Find the half-life of radiocarbon. (ii) Charcoal from an ancient pit contained $\frac{1}{4}$ of the carbon-14 found in living sample of same size. Estimate the age of the charcoal. Sol. (i) $\frac{A_0}{2} = A_0 e^{-(\log 2)\left(\frac{1}{5700}\right)t}$ $\left(\frac{-\log 2}{5700}\right)t = log\left(\frac{1}{2}\right)$ $t = \frac{-5700}{\log 2}(-\log 2) = 5700\ years$ (ii) $\frac{A_0}{4} = A_0 e^{-(\log 2)\left(\frac{1}{5700}\right)t}$ $\left(\frac{-\log 2}{5700}\right)t = log\left(\frac{1}{4}\right)$ $-(log2)t = 5700(log1 - log4)$ $-(log2)t = -2(log2)5700$ $t = 11400\ years$	1 1 1 1 1

Q34. A machine costing Rs. 50,000 depreciates at a constant rate of 8%. What is the depreciation charge for the 8th year. If the estimated useful life of the machine is 10 years, determine its scrap value.

Sol. It is given that C = Rs. 50000 and r = 0.08

The book value at the end of the 7th year= $C(1-r)^7 = 50000\ (0.92)^7 = Rs. 27892.33$ 1+1/2

The book value at the end of the 8th year= $C(1-r)^8 = 50000\ (0.92)^8 = Rs. 25660.94$ 1+1/2

Hence depreciation charge for the 8th year = 27892.33 – 25660.94 = Rs. 2231.39 1

The scrap value of the machine is given by

$S = C(1-r)^{10} = 50000\ (0.92)^{10} = Rs. 21719.42$ 1

Q35. A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for atmost 12 hours whereas machine III must be operated atleast 5 hours a day. He produces only two item M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹600 and ₹ 400 on each item of M and N respectively. How many units of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produces? What will be the maximum profit?

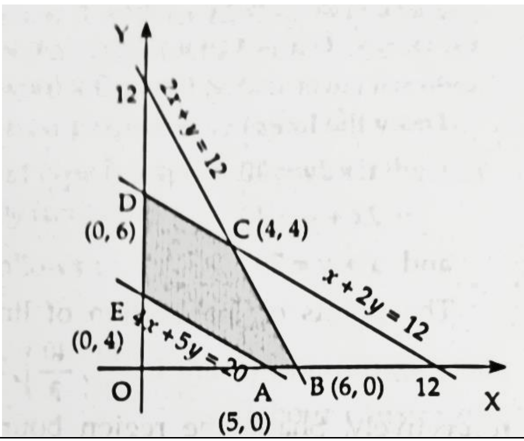
Sol. Let x units of item M and y units of item N are produced by the manufacturer, then the problem can be formulated as an L.P.P. as follows 1

Maximize the profit (in ₹) Z = 600x + 400y subject to the constraints

	$x + 2y \le 12$	(machine I constraint)
	$2x + y \le 12$	(machine II constraint)
	$1x + 1.25y \ge 5$	(machine III constraint)
i.e.	$4x + 5y \ge 20,$	
	$x \ge 0,\ y \ge 0$	(non-negativity constraints)

1

Draw the lines x + 2y = 12, 2x + y = 12 and 4x +5y = 20, and shade the region satisfied by the above inequalities. 1



The feasible region is the polygon ABCDE, which is convex and bounded. 1

The corner points are A (5, 0), B(6, 0), C(4, 4), D(0, 6) and E (0, 4).

The values of Z at the corner points are: 1

at A(5, 0), Z = 600 × 5 + 400 × 0 = 3000;

at B(6, 0), Z = 600 × 6 + 400 × 0 = 3000;

at C(4, 4), Z = 600 × 4 + 400 × 4 = 4000;

at D(0, 6), Z = 600 × 0 + 400 × 6 = 2400;

at E(0, 4), Z = 600 × 0 + 400 × 4 = 1600.

	∴ Maximum profit = ₹4000, when 4 items of M and 4 items of N are produced. **OR** There are two types of fertilizers 'A and B'. 'A' consists of 12% nitrogen and 5% phosphoric acid whereas 'B' consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If 'A' costs 10 per kg and 'B' costs Rs. 8 per kg, then using linear programming determine how much of each type of fertilizer should be used so that the nutrient requirements are met at a minimum cost? Sol. Let the farmer uses x kg of fertilizer A and y kg of fertilizer B. The given data can be summarized as follows:	

	Fertiliser *A*	Fertiliser *B*	Minimum requirement (in kg)
Nitrogen (in%)	12	4	12
Phosphoric acid (in %)	5	5	12
Cost (in ₹/kg)	10	8	

1

1

The inequations thus formed based on the given problem are as follows: 1

$$\frac{12x}{100} + \frac{4y}{100} \geq 12$$

$\Rightarrow 12x + 4y \geq 1200$

$\Rightarrow 3x + y \geq 300$ 1

Also, $\frac{5x}{100} + \frac{5y}{100} \geq 12$

$\Rightarrow 5x + 5y \geq 1200$

$\Rightarrow x + y \geq 240$ 1

and $x \geq 0$, $y \geq 0$

Let Z be the total cost of the fertilisers. Then,

Z = 10x+ 8y

The LPP can be stated mathematically as Minimise Z = l0x + 8y

Subject to constraints $3x + y \geq 300$, $x + y \geq 240$, $x \geq 0$, $y \geq 0$

To solve the LPP graphically, let us convert the inequations into equations as follows:

3x + y = 300 …(i)

x + y = 240 …..(ii)

Table for line 3x + y = 300 is

x	0	100
y	300	0

So, it passes through (0, 300) and (100, 0).

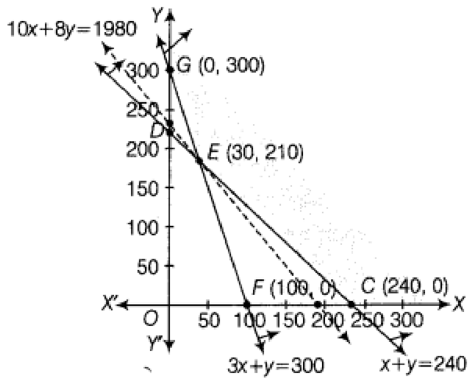
On putting (0,0) in the inequality $3x + y \geq 300$, we get $3(0) + 0 \geq 300 \Rightarrow 0 \geq 300$ (which is false)

So, the half plane is away from origin (1)

Table for line x + y = 240 is

x	0	240
y	240	0

So, it passes through (240, 0) and (0, 240).
On putting (0, 0) in $x + y \geq 240$, we get $0 + 0 \geq 240$ (which is false)
So, the half plane is away from origin.
Also, $x \geq 0$ and $y \geq 0$, so the region lies in 1st quadrant.
The graph of inequations is shown below



The shaded region GEC represents the feasible region of given LPP and it is unbounded.
On solving Eqs. (i) and (ii), we get x = 30 and y = 210
So, the point of intersection is B (30, 210).

Corner Points	Value
G (0,300)	2400
C (240,0)	2400
E (30,210)	1980 – Minimum

From the table, we find that 1980 is the minimum value of Z at E (30, 210). Since, the region is unbounded, therefore, 1980 may or may not be the minimum value of Z. For this we have to check that the open half plane $10x + 8y < 1980$ has any point common or not with the feasible region. Since, it has no point in common with the feasible region. So, the minimum value of Z is obtained at (30, 210) and the minimum value of Z is 1980. So, the farmer should use 30 kg of fertilizer A and 210 kg of fertilizer B.

SECTION – E

(Each case study carries 4-mark weightage)

Q36. **<u>CASE STUDY – I</u>**

A loan of Rs. 2,50,000 at the interest rate of 6% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 5 years.
Based on the above information, answer the following questions. Show steps to support your answers.

(i) Find the size of each monthly payment
(ii) Find total interest paid
(iii) Find the principal outstanding at beginning of 40^{th} month.

OR

Find the interest paid in 40^{th} month.

	(Given $(1.005)^{60} = 1.3489, (1.005)^{21} = 1.1104$) Sol. Given, P = Rs. 250000, $i = \frac{6}{12\times100} = 0.005$ and $n = 5 \times 12 = 60$ (i) $EMI = \frac{250000 \times 0.005 \times (1.005)^{60}}{(1.005)^{60} - 1}$ $= \frac{250000 \times 0.005 \times 1.3489}{0.3489} = Rs. 4832.69$ (ii) Total interest paid = $n \times EMI - P = 60 \times 4832.69 - 250000$ $= 289961.40 - 250000 = Rs. 39961.40$ (iii) Principal outstanding at beginning of 40th month $= \frac{EMI[(1+i)^{60-40+1} - 1]}{i(1+i)^{60-40+1}}$ $= \frac{4832.69 \times [(1.005)^{21} - 1]}{0.005 \times (1.005)^{21}}$ $= \frac{4832.69 \times [1.1104 - 1]}{0.005 \times 1.1104}$ $= \frac{4832.69 \times 0.1104}{0.005 \times 1.1104} = Rs. 96096.72$ **OR** Interest paid in 40th payment = $\frac{EMI[(1+i)^{60-40+1} - 1]}{(1+i)^{60-40+1}}$ $= \frac{4832.69 \times [(1.005)^{21} - 1]}{(1.005)^{21}}$ $= \frac{4832.69 \times 0.1104}{1.1104} = Rs. 480.48$	 1 1 1+1 1+1
Q37.	**<u>CASE STUDY – II</u>** The marginal cost ($in\ Rs.$) of a product is given by $MC = \frac{300}{\sqrt{3x+25}}$ and the fixed cost is $Rs. 5000$. Based on the above information, answer the following questions. Show steps to support your answers. (i) Find the cost function. (ii) Find the average cost function. (iii) Find the cost of producing 25 units of the product. OR (iv) Find the average cost of producing 200 units of the product Sol. (i) $C(x) = 200\sqrt{3x+25} + 4000$ (ii) $AC = \frac{200\sqrt{3x+25}}{x} + \frac{4000}{x}$ (iii) Rs. 6000 **OR** Rs. 45	 1+1 1 1 1

Q38.

CASE STUDY III

A school plans to award Rs. 6000 in total to its students to reward for certain values – honest, regularity and hard work. When three time the award money for hard work is added to the award money given for honesty amounts to Rs. 11,000. The award money for honesty and hard work together is double the award money for regularity. Use matrix method to find the prize money for each category of award.

Sol**.** Let the prize money for honesty = Rs. x
Prize money for regularity = Rs. y
Prize money for hard work = Rs. z

1

ATQ

$x + y + z = 6000$

$x + 3z = 11000$ 1

$x - 2y + z = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$ 1+1

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$|A| = 6$ 1+1

$$adj.A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$X = A^{-1}B$

$$= \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$

$x = 500, y = 2000, z = 3500$

OR

The sum of three numbers is 20. If we multiply the first number by 2 and add the second number to the result and subtract the third number, we get 23. By adding second and third numbers to three times the first number, we get 46. Represent the above problem algebraically and use Cramer’s rule to find the numbers from these equations.

1

Sol. Let the three numbers be x, y and z

ATQ

$x + y + z = 20$ 1

$2x + y - z = 23$

$3x + y + z = 46$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{vmatrix} = -4$$ 1+1

	$D_1 = \begin{vmatrix} 20 & 1 & 1 \\ 23 & 1 & -1 \\ 46 & 1 & 1 \end{vmatrix} = -52$ $D_2 = \begin{vmatrix} 1 & 20 & 1 \\ 2 & 23 & -1 \\ 3 & 46 & 1 \end{vmatrix} = -8$ $D_3 = \begin{vmatrix} 1 & 1 & 20 \\ 2 & 1 & 23 \\ 3 & 1 & 46 \end{vmatrix} = -20$ $\therefore x = 13, y = 2, z = 5$	1+1